Units and Measures in Wolfram|Alpha

Physical dimension-based infrastructure for computing with scientific data

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Scientific and Technical Data in Wolfram|Alpha

Use Data for (Real-Time) Computations

Units, Units, Units, …, Measures, …

Unit Conversions and Orders of Magnitude

Physical Quantities

Generalized Units

Unit and Physical Quantity Uses

Future (of units in Wolfram|Alpha)

Scientific and Technical Data in Wolfram|Alpha

Diverse data sets

‘Object’ data

Real-time data

Physical law data

Mathematical data

…

Data characteristics:

• all data are curated

• all data are properly structured

• interconnection of data

• all data are soon fully 'sliceable'

• potentially large in size (TB of data)

…

• virtually all data are measured in **units**

‘Object’ data

- chemicals (melting points, densities, …)
- materials (conductivities, hardness, …)
- food (calories, vitamin contents, …)
- astronomical objects (distance, mass, …)
- species (sizes, ages, …)
- genetic data (sequences)

…

WolframAlpha computational knowledge engine

boiling point methanol vs propanol

Input interpretation: *Mathematica* form

methanol N–propanol	boiling point

Results:

methanol	64.7 °C (degrees Celsius)
N–propanol	97 °C (degrees Celsius)

Computed by: Wolfram *Mathematica* Source information » Download as: PDF | Live *Mathematica*

Real-time data

- weather (temperatures, air pressures, …)
- earthquakes (magnitudes, wave characteristics, …)
- UV radiation (UV indices, forecasts, …)

…

WolframAlpha™ computational... knowledge engine

recent earthquakes in europe > 4

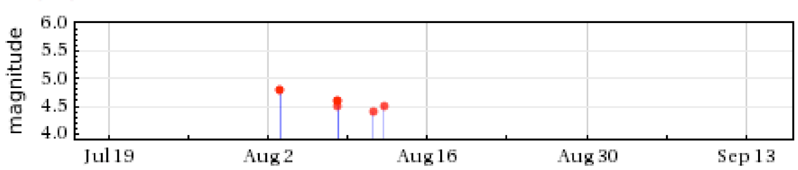
Assuming Europe | Use Europe with Russia and Turkey or Europe with Russia instead

Input interpretation:

earthquakes	Europe	with magnitude > 4

Results: Last 2 months | Magnitude > 4 | CDT

Timeline:



List: Sort by magnitude | More | Show coordinates | Show metric

magnitude	time	location
4.8	Tue, Aug 3, 2010 02:40 am CDT (1.5 months ago)	12 mi ENE of Sardinia, Aitolia Kai Akarnania, Greece
4.6	Sun, Aug 8, 2010 04:00 am CDT (1.3 months ago)	5 mi ESE of Nea Maditos, Thessaloniki, Greece

Physical law data

- effects, principles, laws, paradoxa, formulas, …

- formulas, …

Example: Pendulum

$$T = 2\pi\sqrt{\frac{l}{g}} \qquad f = \frac{1}{T}$$

$$v_{max} = \sqrt{2\, g\, l\, (1 - \cos(\theta_0))}$$

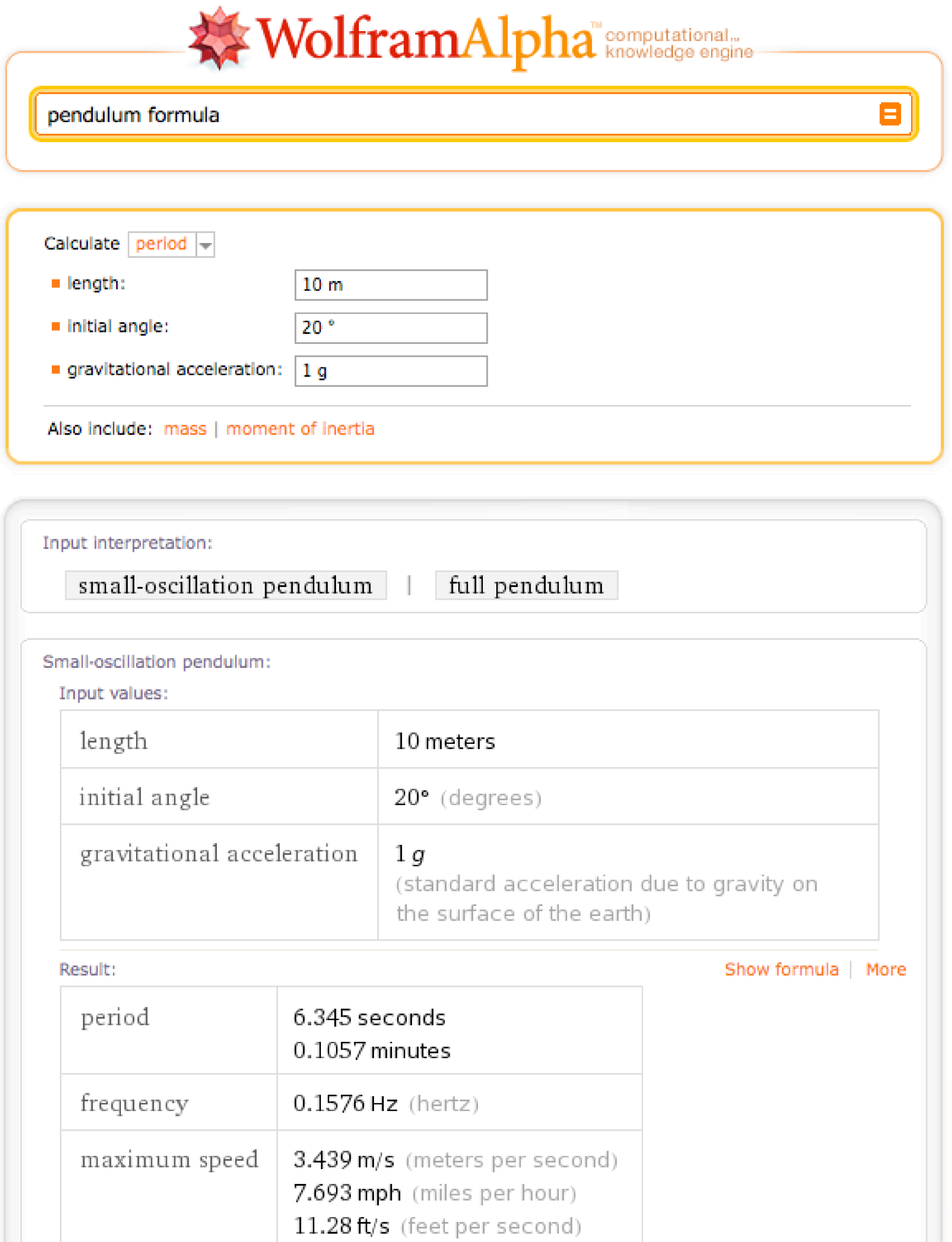
T	period
l	length
θ_0	initial angle
g	gravitational acceleration
f	frequency
v_{max}	maximum speed

⟺

$$T = 4\sqrt{\frac{l}{g}}\, K\left(\sin^2\left(\frac{\theta_0}{2}\right)\right) \qquad f = \frac{1}{T}$$

$$v_{max} = \sqrt{2\, g\, l\, (1 - \cos(\theta_0))}$$

T	period
l	length
θ_0	initial angle
g	gravitational acceleration
f	frequency
v_{max}	maximum speed


WolframAlpha computational knowledge engine
pendulum formula
Calculate period
length: 10 m
initial angle: 20 °
gravitational acceleration: 1 g
Also include: mass | moment of inertia
Input interpretation:
small-oscillation pendulum | full pendulum
Small-oscillation pendulum:
Input values:
length 10 meters
initial angle 20° (degrees)
gravitational acceleration 1 g (standard acceleration due to gravity on the surface of the earth)
Result:
Show formula | More
period 6.345 seconds 0.1057 minutes
frequency 0.1576 Hz (hertz)
maximum speed 3.439 m/s (meters per second) 7.693 mph (miles per hour) 11.28 ft/s (feet per second)

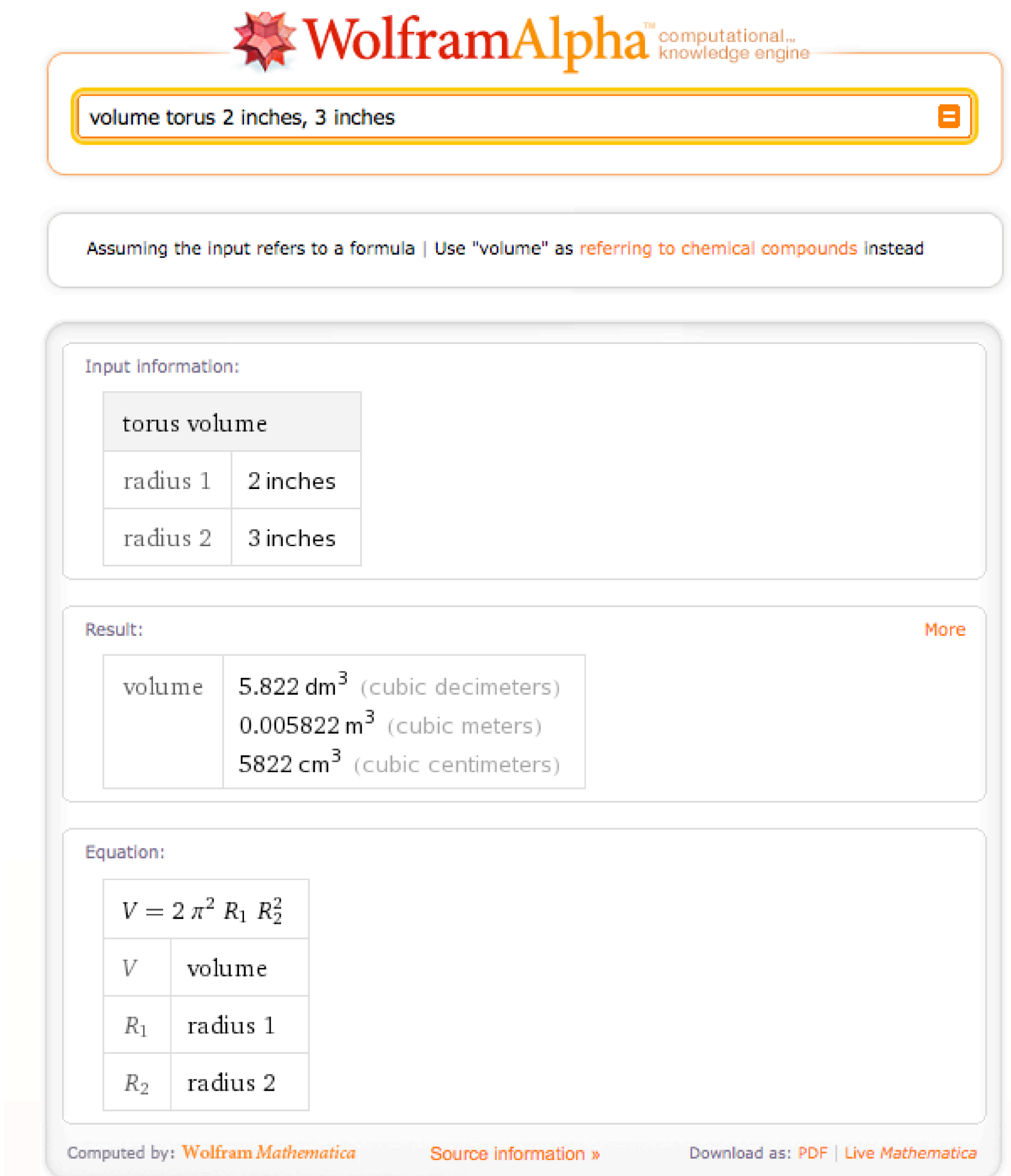
Mathematical data

- named polyhedra, curves, knots, graphs, curves, surfaces, lamina, … (areas, volumes, invariant polynomials, …)

- mathematical identities (values, representations of special functions [$J_\nu(z)$, $P_n^m(x)$, ${}_2F_1(a, b, c; z)$,…] and mathematical constants [π, e, γ, …])

- mathematical conjectures, theorems, axioms

Even mathematical data need sometimes units, e.g. geometrical data



Astronomical object with "exotic" units

WolframAlpha computational knowledge engine

crab pulsar

Input interpretation: *Mathematica form*

Crab Pulsar (pulsar)

Basic properties:

alternate names	B0531+21 | J0534+2200 | PSR B0531+21 | PSR J0534+2200
discovery year	1968 (42 years ago)
pulsar type	spin–powered pulsar with pulsed emission from radio to infrared or higher frequencies
surveys	Arecibo Survey 4 | Green Bank Fast Pulsar Survey

Distance and motion properties:

distance from Earth	6519 ly 1.999 kpc
proper motion right ascension	–14.7 mas/yr
proper motion declination	2 mas/yr
transverse velocity	140.67 km/s

Units »

Rotation properties: More

rotation period	0.03308471603 s (barycentic)
rotation frequency	2.61147776×10^6 rev/day (barycentic) 2.60434756×10^6 angular speeds of the earth's rotation on its axis
rotation measure	$-42.3\ \text{rad/m}^2$
spin down age	1240 yr
spin down age corrected for proper motion	1240 yr
glitch count	12 (in 42 years)

rotation period | 0.0330847

Units »

Radiation properties: More

pulse width at 50% peak	3 ms
pulse width at 10% peak	4.7 ms
spin down energy loss rate	4.6×10^{31} W
dispersion measure	56.791 pc/cm^3
temporal broadening	1.51×10^{-6} s
energy flux measured at Earth	1.2×10^{38} erg/(kpc^2 s)
magnetic field at light cylinder	98 T 980 000 G
mean flux density at 400 MHz	646 mJy
mean flux density at 1400 MHz	14 mJy
surface magnetic dipole corrected for proper motion	3.78×10^{8} T 3.78×10^{12} G
surface magnetic flux density	3.78×10^{8} T 3.78×10^{12} G
radio luminosity 400 MHz	2584 mJ kpc^2
radio luminosity 1400 MHz	56 mJ kpc^2
measured spectral index	−3.1

Units »

Use Data for (Real-Time) Computations

Why not just store everything?

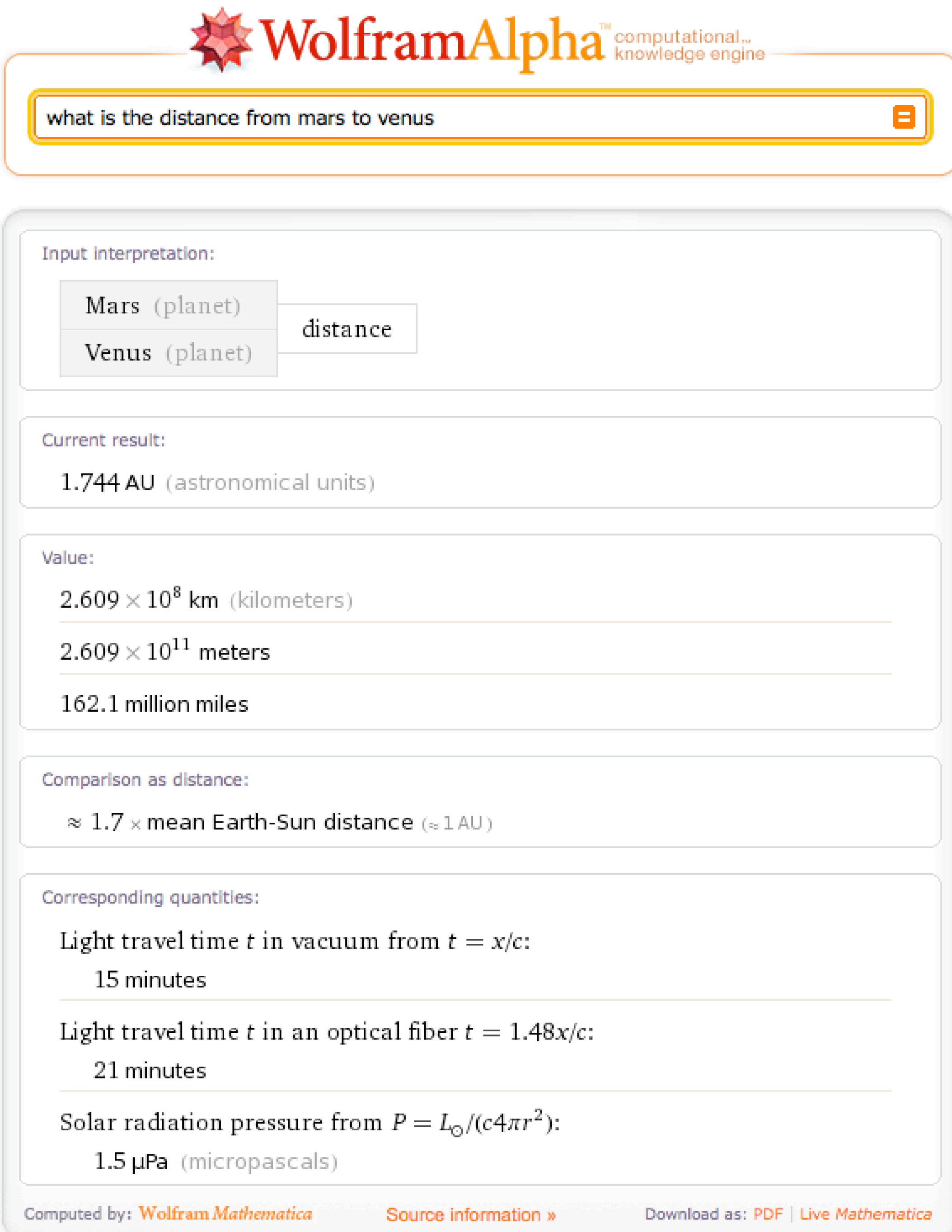
- time-dependent data
- combinatorial explosion of product sets
- free parameters in applications

Calculating with data $\Longleftarrow$ calculating with units

Real-time computed data

- astronomical objects
- geo-distances

…

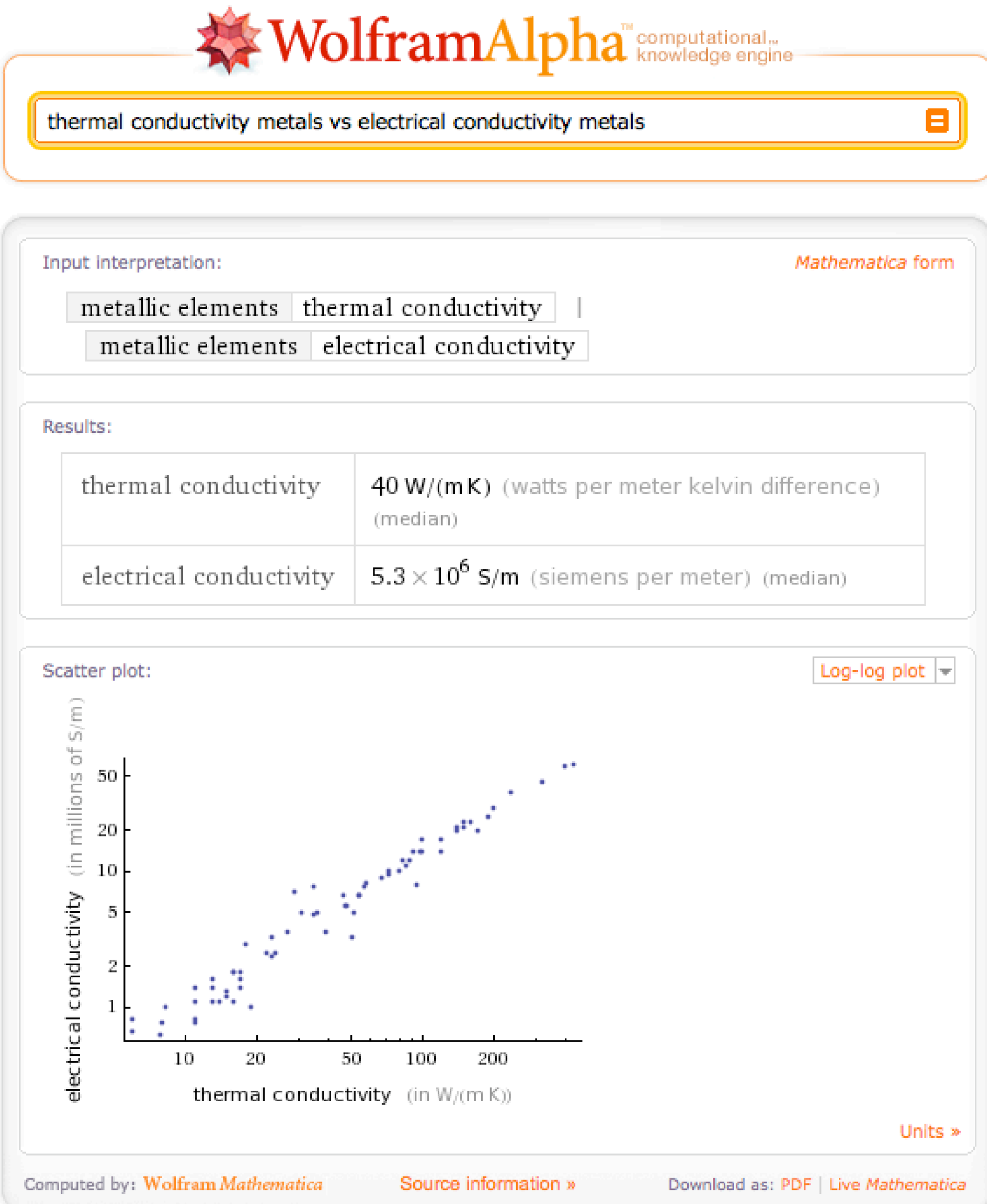


Data analysis

- averages, distributions
- correlations
- genetic data look-ups

…

Wiedemann-Franz law



Units, Units, Units, …, Measures, …

All that can be measured must be measured in reference to some agreed-on yardstick.

Calculations with data require calculations with units.

● length, time, area, volume

● pressure, force, energy, electric current, …

● since 1790: International System of Units

- seven base units: meter, second, kilogram, kelvin, ampere, candela, mole (radian, steradian)

Most countries use the SI system today.

Exceptions: 

To deal with diverse data, one needs to handle units well!

Wolfram|Alpha knows tens of thousands of units

Knowing a unit means:
- numerical values and precision (c is exact, trusted digits of $\hbar$)
- relations to other units
- typical uses
- names, abbreviations, …
- formatting

…

Unit Conversions and Orders of Magnitude

- default conversions
- default comparisons
- country-specific preferred units

WolframAlpha computational knowledge engine

200 US fluid ounces

Input interpretation:

200 fl oz (fluid ounces)

Unit conversions: More

1.563 gallons

6.25 quarts

12.5 pints

25 cups

360.9 in^3 (cubic inches)

Comparisons as volume:

≈ 0.26 × molar volume (1 mol molar volume constants)

≈ volume of blood in a typical human (≈ 5 L)

Comparisons as volume of food or beverage:

≈ 1.9 × US can size 10 (105 fl oz)

≈ 3.6 × US can size 5 (7 cups)

≈ 6.2 × US can size 3 (4 cups)

Interpretations: More

volume

volume of food or beverage

Basic unit dimensions:

$[\text{length}]^3$

Corresponding quantities:

Radius r of a sphere from $V = 4\pi r^3/3$:

0.3681 feet
4.417 inches
11.22 cm (centimeters)

Edge length a of a cube from $V = a^3$:

0.5933 feet
7.12 inches
18.08 cm (centimeters)

Mass m of water from $m = \rho V$:

13 lb (pounds)
5.9 kg (kilograms)
(assuming water density ≈ 1000 kg/m^3)

Molecules N of water from $N = \rho V/\text{MW}$:

1.977×10^{26} molecules
(assuming water density ≈ 1000 kg/m^3)

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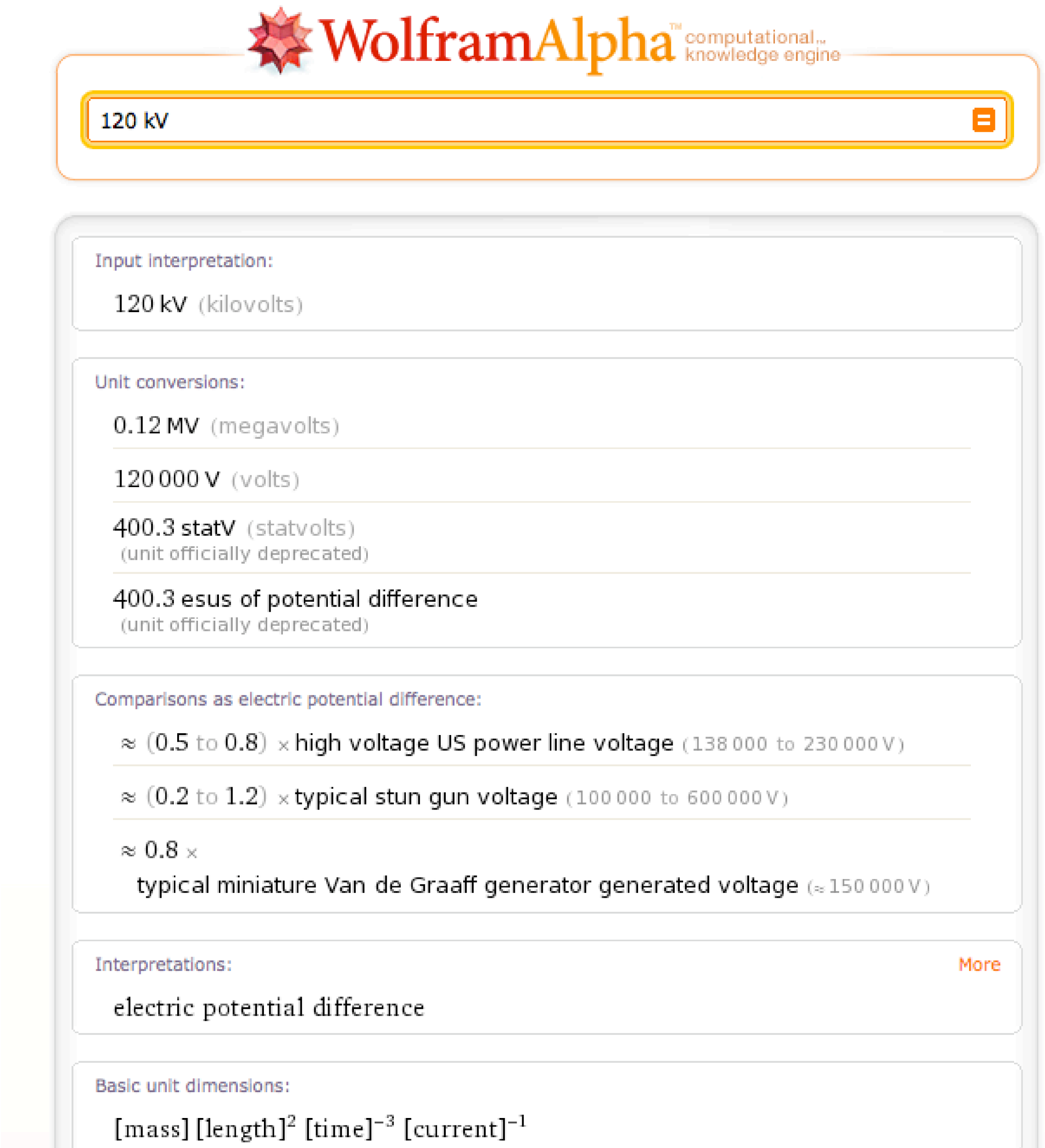
WolframAlpha computational knowledge engine
120 kV
Input interpretation:
120 kV (kilovolts)
Unit conversions:
0.12 MV (megavolts)
120 000 V (volts)
400.3 statV (statvolts)
(unit officially deprecated)
400.3 esus of potential difference
(unit officially deprecated)
Comparisons as electric potential difference:
≈ (0.5 to 0.8) × high voltage US power line voltage (138 000 to 230 000 V)
≈ (0.2 to 1.2) × typical stun gun voltage (100 000 to 600 000 V)
≈ 0.8 × typical miniature Van de Graaff generator generated voltage (≈ 150 000 V)
Interpretations:
More
electric potential difference
Basic unit dimensions:
[mass] [length]^2 [time]^-3 [current]^-1

Corresponding quantities:

Radius r of a 1 meter long copper wire carrying 1 ampere current from $\Delta\phi = I\rho L/(\pi r^2)$:

2.1×10^{-7} meters

(assuming resistivity of copper ≈ 1.72 × 10

Radius r of a 1 meter long copper wire carrying 1 amp
= IrhoL/(pir^2):...

Relativistic velocity v of an electron after traversal of this potential difference $m_0 \gamma v^2/2 = e\Delta\phi$:

$0.61\,c$ (speeds of light)

Energy E from $E = e\Delta\phi$:

120 keV (kiloelectronvolts)

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WolframAlpha computational knowledge engine

4 arpents

Assuming Louisiana arpent areas for "arpents" | Use Arkansas arpent areas or French arpent lengths instead

Input interpretation:

4 Louisiana arpent areas

Unit conversions:

147 456 ft^2 (square feet)

0.0137 km^2 (square kilometers)

13 699 m^2 (square meters)

1.37 hectares

3.385 acres

Comparisons:

≈ (0.25 ≈ 1/4) × area of the base of the Great Pyramid of Giza (≈ 55 000 m^2)

≈ 1.9 × area of a FIFA–sanctioned international match soccer field (7700 to 9600 yd^2)

≈ (1 to 3) × area of a FIFA–sanctioned soccer field (5000 to 13 000 yd^2)

Interpretation:

area

Basic unit dimensions:

$[\text{length}]^2$

Corresponding quantities:

Radius r of a circle from $A = \pi r^2$:

216.6 feet

66.03 meters

Radius r of a sphere from $A = 4\pi r^2$:

108.3 feet

33.02 meters

Edge length a of a square from $A = a^2$:

384 feet

0.07273 miles

117 meters

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WolframAlpha computational knowledge engine

12.3 queenquoons

Input interpretation:

12.30 QQn (queenquoons)

Unit conversions: More

1.971×10^{18} C (coulombs)

1.971×10^{17} abC (abcoulombs)
(unit officially deprecated)

5.908×10^{27} statC (statcoulombs)
(unit officially deprecated)

5.908×10^{27} Fr (franklins)

5.908×10^{27} esus of charge
(unit officially deprecated)

Interpretations: More

electric charge

Basic unit dimensions:

[time] [current]

Corresponding quantities:

Current I caused by the charge flowing in one second from $I = Q/t$:

1.971×10^{18} A (amperes)

Force F between like charges 1 cm apart from $F = Q^2/(4\pi\epsilon_0 r^2)$:

3.49×10^{50} N (newtons)

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Physical constants (CODATA)

WolframAlpha computational knowledge engine

speed of light

Input interpretation:

c (speed of light in vacuum)

Value: Show exact value

299 792 km/s (kilometers per second)

2.998×10^{8} m/s (meters per second)

186 282 mi/s (miles per second)

6.706×10^{8} mph (miles per hour)

1 Planck speed

Comparison:

$\approx 2.4 \times$ speed of light in diamond ($\approx 1.24 \times 10^{8}$ m/s)

Interpretation:

speed

Basic unit dimensions:

$[\text{length}]\,[\text{time}]^{-1}$

Corresponding quantity:

Slowness from $S = 1/v$:

3.336×10^{-9} s/m (seconds per meter)

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Calculations with units

WolframAlpha computational... knowledge engine

0.008 (electron charge)^3/hbar

Assuming multiplication | Use a list instead

Assuming reduced Planck constants for "hbar" | Use hectobars instead

Input interpretation:

$0.008 \frac{(e \text{ (electron charge)})^3}{\hbar \text{ (reduced Planck constant)}}$

Result:

$3.12 \times 10^{-28}\ s^4 A^3/(g\,m^2)$ (second to the fourth amperes cubed per gram meter squared)

Unit conversions:

1.947×10^{-6} A eV/V^2 (ampere electronvolts per volt squared)

3.12×10^{-25} A J/V^2 (ampere joules per volt squared)

1.263 a_{FN} (first Fowler–Nordheim constants)

Interpretation:

first Fowler–Nordheim constant unit

Basic unit dimensions:

$[\text{mass}]^{-1}\ [\text{length}]^{-2}\ [\text{time}]^{4}\ [\text{current}]^{3}$

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Physical Quantities

Length, time, area, volume, pressure, force, energy, …

Wolfram|Alpha knows thousands of physical quantities

- typical units (N·m as torque vs energy; eV versus BTU)
- relation between physical quantities

WolframAlpha computational knowledge engine

pressure

Assuming "pressure" is a physical quantity | Use as a formula or a weather property or a word or a financial entity instead

Input interpretation:

physical quantity	pressure

Basic dimensions:

$[\text{mass}]\,[\text{length}]^{-1}\,[\text{time}]^{-2}$

Standard unit for pressure:

Pa (pascal)

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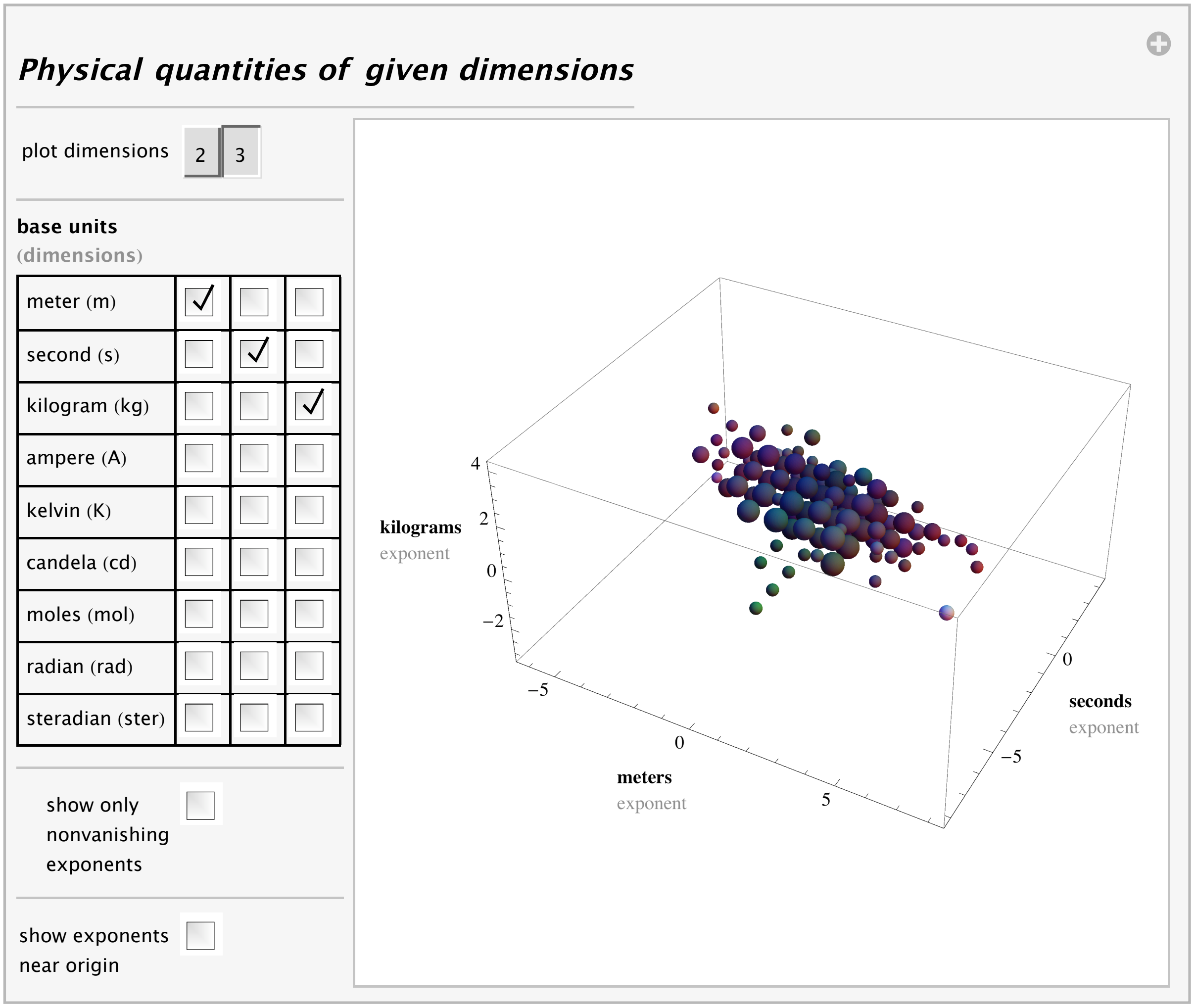
Bird's-eye view on the space of physical quantity dimensions



Dimensional analysis

Taylor’s estimation of the energy released in the first detonated atomic bomb

WolframAlpha computational knowledge engine

pressure, mass density, energy, time, length

Input interpretation:

physical quantity pressure | physical quantity mass density | physical quantity energy | physical quantity time | physical quantity length

Basic dimensions:

pressure	[mass] [length]$^{-1}$ [time]$^{-2}$
mass density	[mass] [length]$^{-3}$
energy	[mass] [length]^{2} [time]$^{-2}$
time	[time]
length	[length]

Standard units:

pressure	Pa (pascal)
mass density	kg/m^3 (kilogram per cubic meter)
energy	J (joule)
time	second
length	meter

Dimensionless combinations:

$$\frac{[\text{length}]^3\,[\text{pressure}]}{[\text{energy}]}$$

$$\frac{[\text{pressure}]^5\,[\text{time}]^6}{[\text{energy}]^2\,[\text{mass density}]^3}$$

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Generalized Units

Real world data contain often more than just units that can be reduced to the seven (nine) SI units. They also contain:

Currencies

Counting units

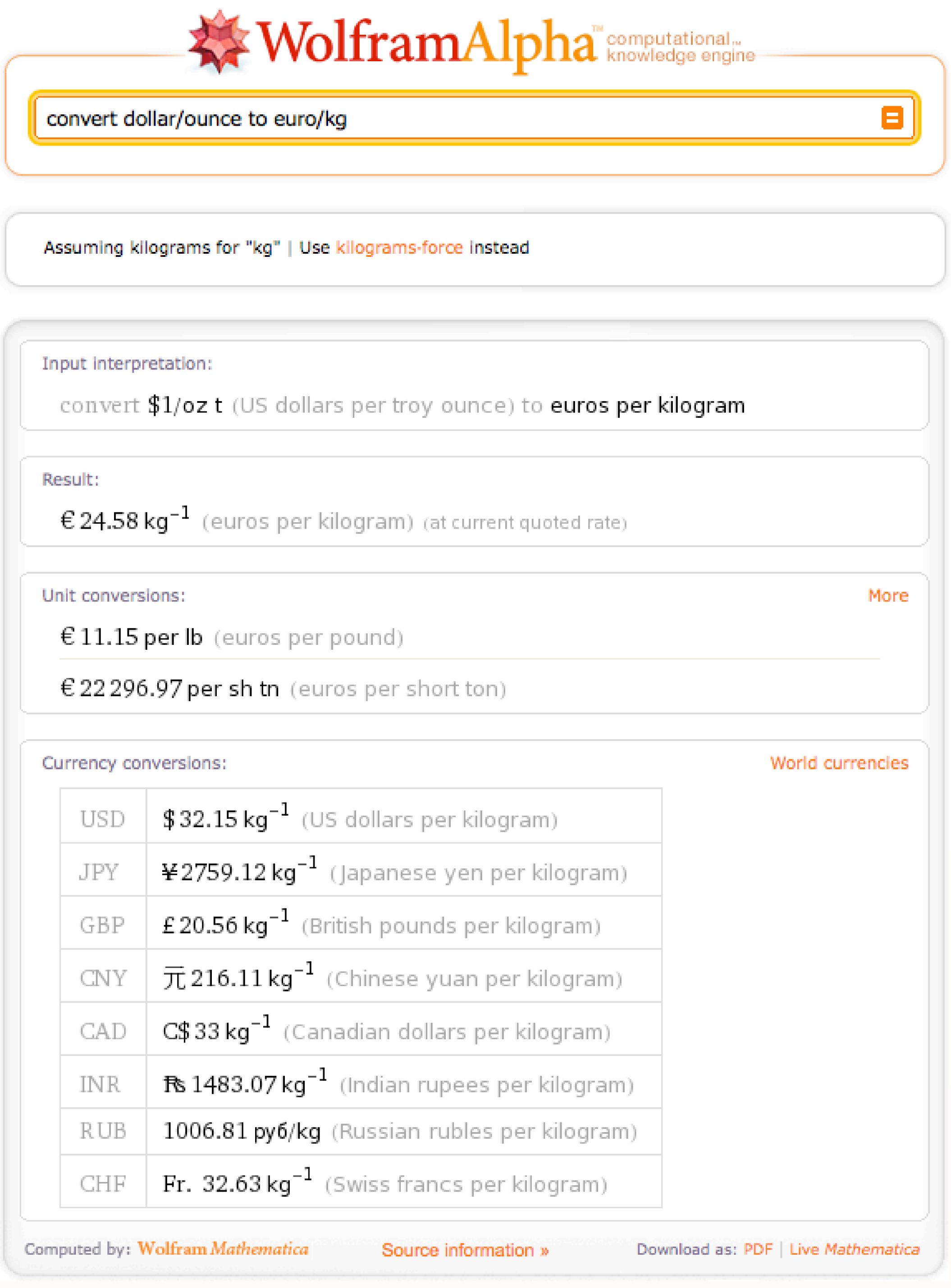
Commodities

Substances

Scales and Industrial measures

Currencies

- $, €, ¥, … (e.g. $/kg)
- time-dependent conversion factors



Counting units

- people, women, men, marriages, boys, girls, children, teenagers, … (e.g. marriages per year)
- samples, pages, …


WolframAlpha computational knowledge engine

400 pages

Input interpretation:

400 generic pages (in English)

Conversions:

1.22×10^6 characters (1.22 MB (megabytes)) (assuming 5.1 characters per word and 1 space between each word)
200 000 words
18 000 lines

Times: More

typical typing	56 hours
typical speaking	22 hours
silent reading	12 hours

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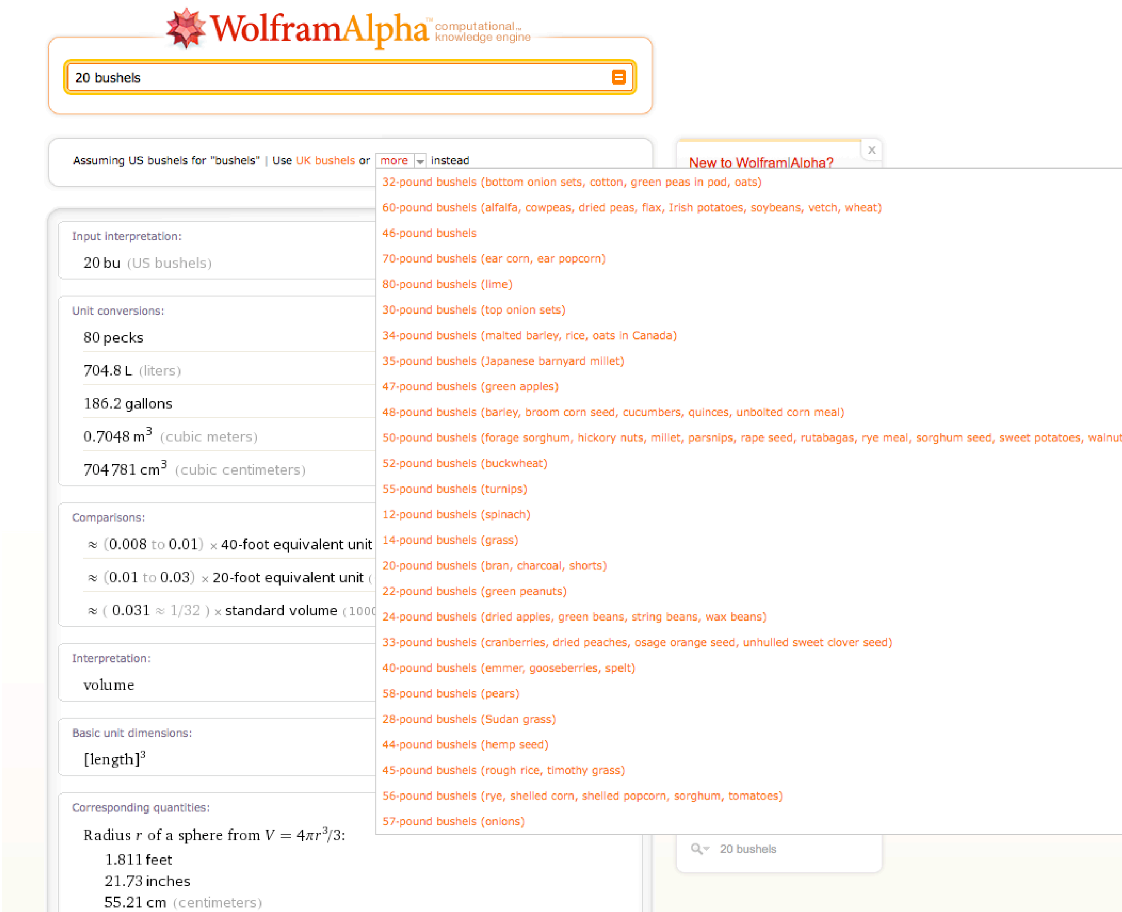
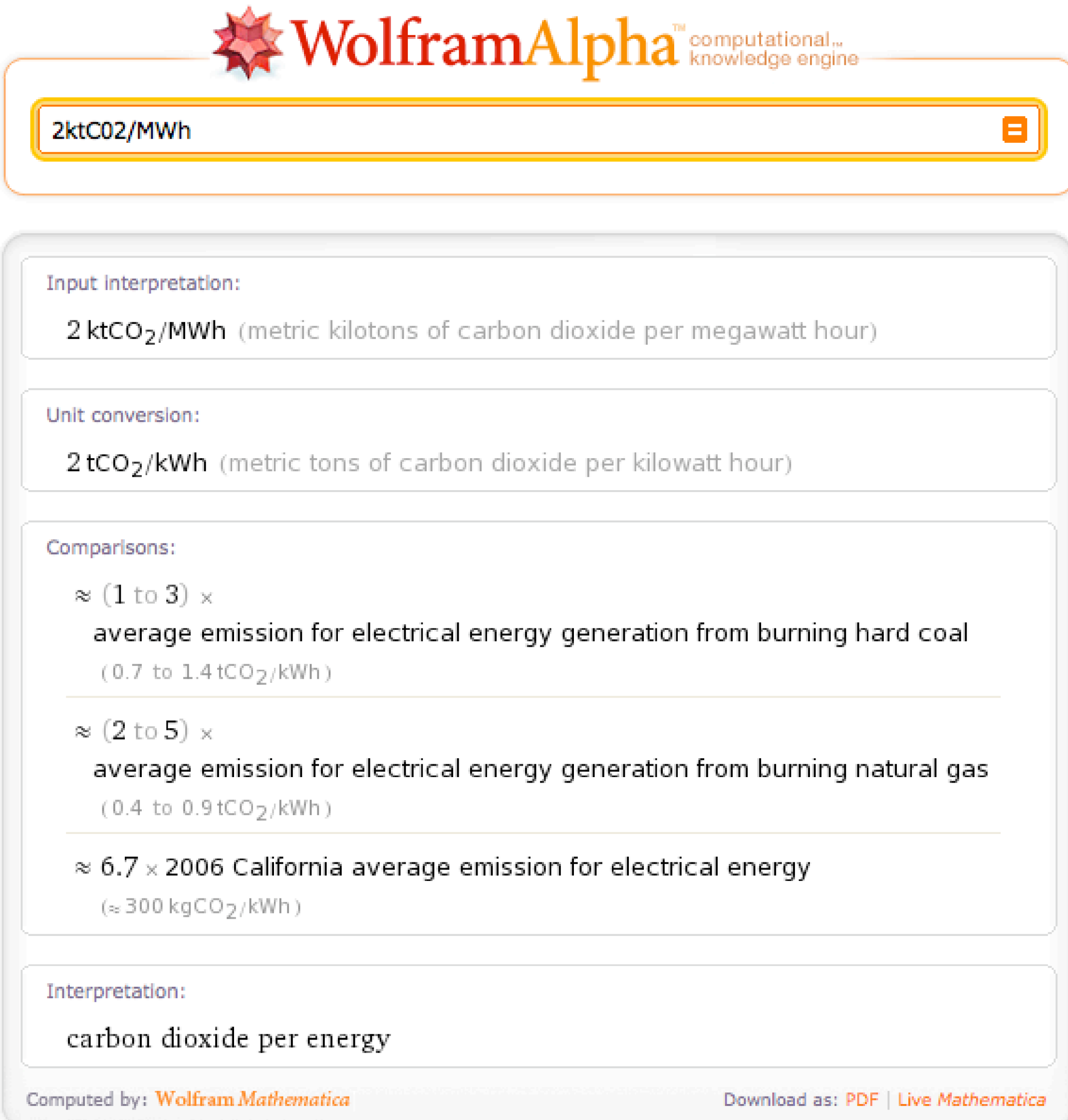
Commodities

- bushels, …



Substances

- CO_2, CH_4, … (e.g. MtC02/MWh) [noncommutative conversions]



Scales and Industrial measures

• Beaufort scale, earthquake magnitudes, screw sizes, …

WolframAlpha computational knowledge engine

seismic moment magnitude 8

Calculate equivalent energy

Input information:

seismic moment magnitude	
seismic moment magnitude	8

Seismic moment magnitude: Show formula | More

equivalent energy	4.2×10^{18} J (joules) 4.2×10^{25} ergs 2.6×10^{37} eV (electronvolts)
equivalent mass of TNT	1×10^{9} metric tons 1 Gt (metric gigaton) 1000 Mt (metric megatons)
seismic moment	1.3×10^{21} J (joules) 1.3×10^{28} ergs 7.9×10^{39} eV (electronvolts)

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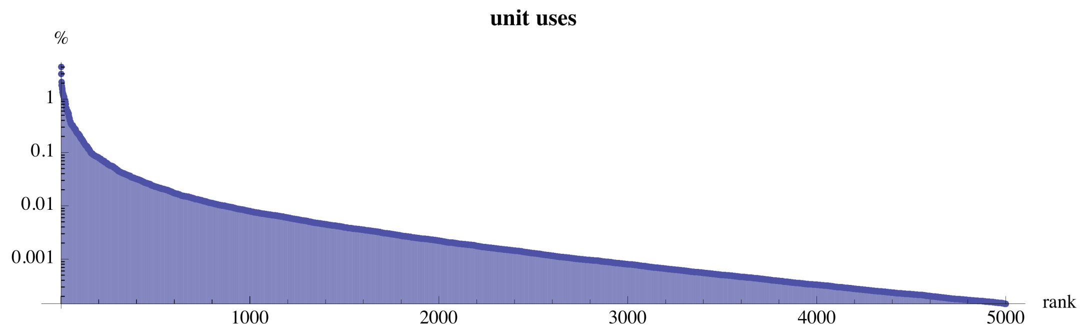
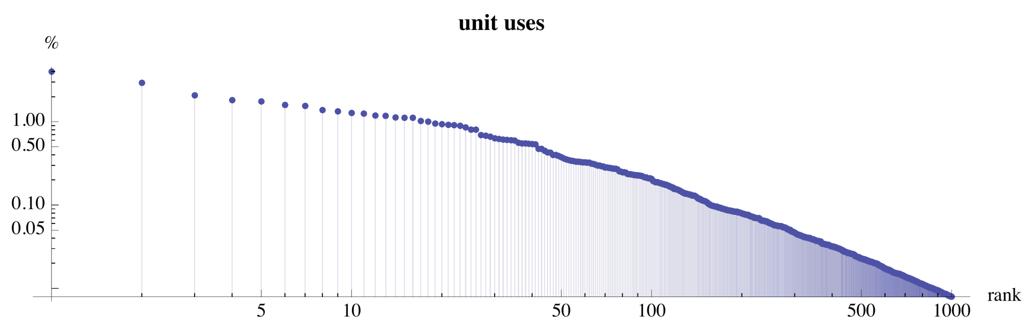
Unit and Physical Quantity Uses

Overall unit uses

Often users have their own unit preferences. (country specific)



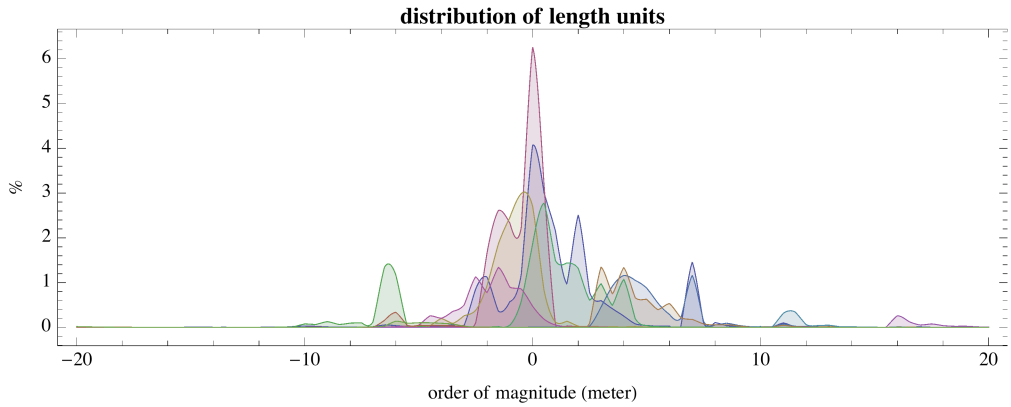
approximate Zipf law

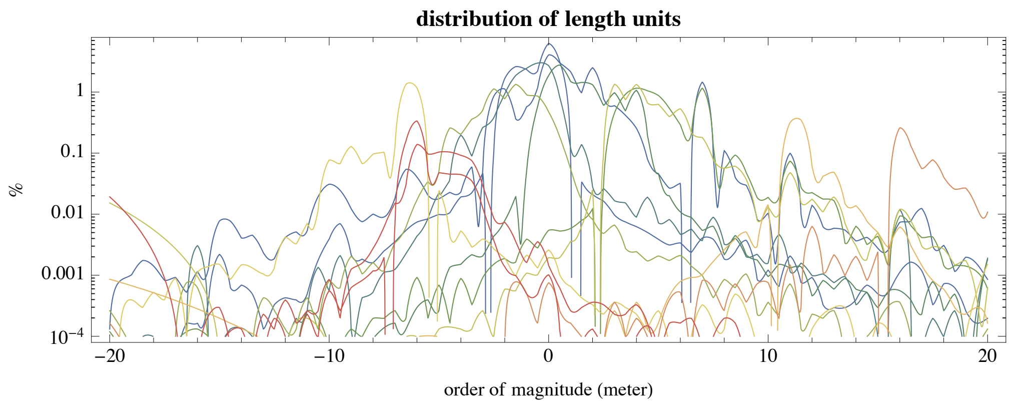


Overall physical quantity uses

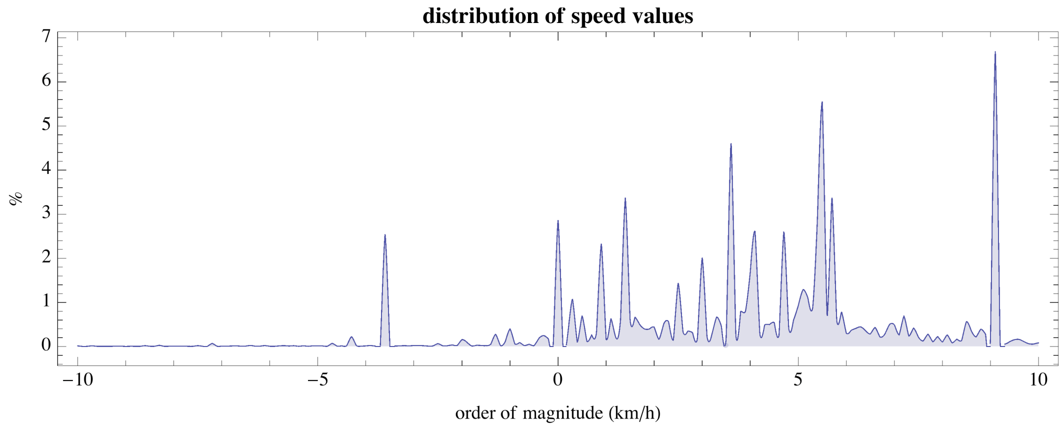
physical quantity	**percentage**
length	30.00
time	27.00
volume	11.00
speed	8.80
temperature	6.10
frequency	3.50
plane angle	3.40
area	3.10
electric current	1.30
amount	0.88
acceleration	0.71
molarity	0.66
electric charge	0.56
voltage	0.52
slowness	0.46
flow	0.42
angular velocity	0.28

Scale dependence of length unit uses





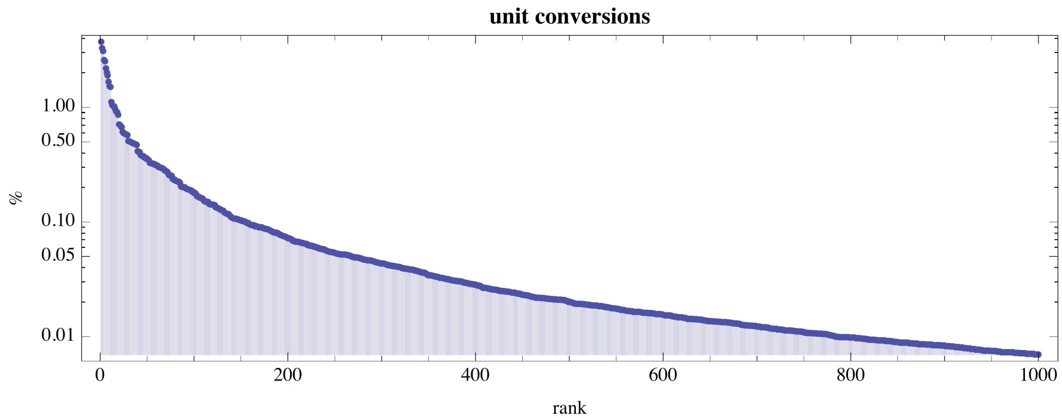
Speed units



Anthropological values

speed value	percentage
75 miles per hour	5.1
340.3 meters per second	4.4
10 meters per second	4.0
88 miles per hour	3.1
1 meters per second	2.7
10 centimeters per second	2.5
4 miles per hour	2.2
150 meters per minute	2.1
45 miles per hour	2.0
30 miles per hour	2.0
72 miles per hour	1.7
46 kilometers per hour	1.4
5 meters per second	1.2
$3.\times 10^7$ meters per second	1.2
340.29 meters per second	1.2
2.5 meters per second	1.0
1.25 meters per second	1.0
70 miles per hour	1.0
60 miles per hour	0.9
15 miles per hour	0.9

Unit conversions



Future (of units in Wolfram|Alpha)

- International units
  - Japanese, Portuguese, …
- Historical units (what is their "exact" value?)
  - Babylonian, Egyptian, …
  - Medieval European units

    (in 1790 about 40,000 different volume, mass, and length measures were used in France)
  - realization changes over time; soon all quantum-based (1m in 1791 = 0.9998 m in 2010)
- Generalized conversions
  - cm → eV, kg → eV (multiply by $\hbar^{\alpha} c^{\beta} G^{\gamma} \ldots$)
  - pint → kg (for sugar, milk, …)
- Unit system
  - cgs, cgs-esu, cgs-emu
  - Atomic, Planck, Stoney, Astronomical, …